Study of Inductive-Capacitive Series Circuits
Using the Simulink Software Package

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1. Introduction

This chapter presents a modern method for approaching the electrical circuits using the MATLAB-SIMULINK package programs. The simple series circuits which are switched on a DC voltage at the initial moment are presented below. For the young researchers these are very useful. We can determine the current variation forms and the reactive elements voltage, by using this virtual medium. Each presented case contains an analytical presentation of the problem, but it also contains electrical diagrams of electrical parameters. The diagrams were obtained by different methods which use this programs package.

2. Study of the inductive series circuits

We will consider the RL series circuit with concentrated parameters from Fig.1. At the initial moment, the k circuit switcher is closed and we intend to study the behavior and variation of circuit electrical parameters after connection.

![RL series circuits at closing](image)

Fig. 1. RL series circuits at closing

2.1 Theoretical study of the circuit

The differential equation which corresponds to the transitory regime immediately after closing is the following:

\[ Ri + L \frac{di}{dt} = E \] (1)
The current expression through the circuit after closing is the solution of the differential equation of the circuit (1).

\[ i(t) = \frac{E}{R} \left( 1 - e^{-\frac{t}{T}} \right) \]  

(2)

where:

\[ T = \frac{L}{R} \]  

(3)

and it represents the time constant of the circuit.

The voltage expression on the coil after connection is:

\[ u(t) = L \frac{di}{dt} = E \cdot e^{-\frac{t}{T}} \]  

(4)

2.2 SIMULINK model of the circuit

The SIMULINK model of the circuit after closing was done on grounds of equation (1) where the derivative of the current was separated.

\[ \frac{di}{dt} = \frac{1}{L} (E - Ri) \]  

(5)

The SIMULINK model obtained in this way is shown in Fig. 2 and was created in order to allow the drawing of the current diagrams through the circuit and the voltage on the coil, for different values of the R resistance and the E D.C. voltage.

![SIMULINK model of the RL circuit at closing](image)

Fig. 2. SIMULINK model of the RL circuit at closing

The k switcher is realized using a voltage step signal which is applied at the terminal of the circuit, to simulate the closing of the k switcher.

If we consider the following values of the electric parameters

- D.C. voltage applied: \( E = 24[V] \);
The resistance value: $R = 50\, \Omega$;
The circuit inductivity: $L = 1\, [H]$,
we will obtain the diagrams from Fig.3 and Fig.4.

**Fig. 3. Variation of the coil voltage after closing**

**Fig. 4. Variation of the circuit current**

### 2.3 Analysis of RL series circuits using the SimPowerSistems software package

The SimPowerSystems software package is a component of the MATLAB program which allows analyzing an electrical circuit by drawing it in an editing window. The SimPowerSystems model is presented in Fig.5 and allows the visualization of the circuit current and the coil voltage if we apply a step voltage signal and a unitary impulse. The electrical parameters of the circuit have the values:
R = 50[Ω]; L = 1[H].

Fig. 5. SimPowerSystems model of an inductive circuit

The Continuous powergui analyze block is required to be present in the drawing window for launching the simulation model. We will open an analysis window by double-clicking on this block, Fig. 6.

Fig. 6. The analysis window of the circuit

The selection of the Use LTI Viewer button will lead to the opening of the window in Fig. 7, from which we can select the current diagram or the coil voltage diagram.
The selection of $U_{\text{Voltage Measurement}}$ or $I_{\text{Current Measurement}}$ is followed by the obtaining of the coil voltage diagram and of the circuit current upon applying the step signal and a unitary impulse. These are presented in Fig.8 and Fig.9.
Fig. 9. The current variation upon applying a step signal and a unitary impulse.

We can use the Edit/Plot Configurations and Edit/ Viewer Preferences options, in order to create these diagrams. We can also select the Bode option from these menus, which allows the obtaining of the current frequency diagrams of the RL series circuit.

Fig. 10. The amplitude frequency diagram of the voltage coil.
3. Study of the capacitive series circuits

We will consider the RC series circuit with concentrated parameters from Fig.12. At the initial moment, the k circuit switcher is closed and we want to study the current variation through the circuit and the capacitor voltage variation, after connection.

Fig. 12. RC series circuit at closing

3.1 Theoretical study of the circuit

The integral equation which corresponds to the transitory regime immediately after closing is given by:

\[
Ri + \frac{1}{C} \int idt = E
\]  

Equation (6) is equivalent with differential equation:
The expression of the capacitor voltage after closing is the solution of differential equation (7):

\[ u(t) = E(1 - e^{-\frac{t}{T}}) \]  

(8)

where \( T = RC \), represents the time constant of the circuit.

The circuit current after closing is given by:

\[ i = C \frac{du}{dt} = \frac{E}{R} e^{-\frac{t}{T}} \]  

(9)

### 3.2 SIMULINK model of the circuit

The SIMULINK model of the circuit after closing was done on grounds of equation (7) where the voltage derivative was separated.

\[ \frac{du}{dt} = \frac{1}{RC}(E - u) \]  

(10)

The SIMULINK model obtained is shown in Fig.13 and was created in order to allow the drawing of the current diagrams through the circuit and the voltage capacitor, for different values of the R resistance, of the E D.C. voltage and of the C capacitor.

Fig. 13. SIMULINK model of the RC circuit at closing

The k switcher is realized using the same voltage step signal which is applied at the terminal of the circuit, to simulate the closing of the k switcher.

If we consider the following values of the electric parameters:
- D.C. voltage applied: $E = 24[V]$;
- The circuit resistance: $R = 50[\Omega]$;
- The circuit capacity: $C = 100[\mu F]$,
we will obtain the diagrams from Fig.14 and Fig.15.

![Image of Scope U](image1.png)

**Fig. 14. The capacitor voltage variation**

![Image of Scope I](image2.png)

**Fig. 15. The current variation after closing**

### 3.3 Analysis of RC series circuits using the SimPowerSistems software package

The SimPowerSystems model is presented in Fig.16 and allows the visualization of the circuit current and the capacitor voltage if we apply a step voltage signal and a unitary impulse.
Fig. 16. SimPowerSystems model of the capacitive circuit

The electrical parameters of the circuit have the values:
- \( R = 50[\Omega] \); \( C = 10[\mu F] \).

To study the voltage variation on the capacitor, we follow the steps:

- Select by double clicking the Continuous powergui block from the simulation model.
- Select Use LTI Viewer in the new window;
- We will select U Voltage Measurement.

This method allows us to obtain the voltage capacitor diagram upon applying the step signal and the unitary impulse (Fig.17).

The current variation through the circuit upon applying a step signal and a unitary impulse was obtained by selecting I_Current Measurement option (Fig.18).

Fig. 17. The capacitor voltage variation upon applying a step signal and a unitary impulse.
Fig. 18. The current variation upon applying a step signal and a unitary impulse.

The Bode diagrams which allow the visualization of the frequency behavior of the circuit are presented bellow:

Fig. 19. The capacitor voltage frequency diagrams
4. Study of the inductive-capacitive series circuits

We will consider the RLC series circuit with concentrated parameters from Fig. 21. At the initial moment, the k circuit switcher is closed and we intend to study the behavior and variation of circuit electrical parameters after connection.

\[ Ri + L \frac{di}{dt} + \frac{1}{C} \int idt = E \]  \hspace{1cm} (11)

Fig. 21. RLC series circuit at connecting

4.1 Theoretical study of the circuit

The integral-differential equation which corresponds to the transitory regime of the considered circuit is the following:
or:

\[ LC \frac{d^2 u}{dt^2} + RC \frac{du}{dt} + u = E \]  \hspace{1cm} (12)

The following notations are made:

\[ \delta = \frac{R}{2L} \] \hspace{1cm} (13)

The circuit amortization:

\[ \omega_0 = \frac{1}{\sqrt{LC}} \] \hspace{1cm} (14)

the circuit personal pulsation:

\[ \omega = \sqrt{\delta^2 - \omega_0^2} \] \hspace{1cm} (15)

We will consider the situation when \( \delta > \omega_0 \) or \( R < 2\sqrt{L/C} \), which will be checked by the resistor in the circuit.

Consequently, the solving of the differential equation gives the following solutions:

\[ u(t) = E \left[ 1 - \frac{1}{2\sqrt{\delta^2 - \omega_0^2}} (r_1 e^{r_1 t} - r_2 e^{r_2 t}) \right] \] \hspace{1cm} (16)

\[ i(t) = \frac{E}{2L\sqrt{\delta^2 - \omega_0^2}} (e^{r_1 t} - e^{r_2 t}) \] \hspace{1cm} (17)

where \( r_1 \) and \( r_2 \) are the roots of the characteristic equation:

\[ r_1 = -\delta + \omega \]
\[ r_2 = -\delta - \omega \] \hspace{1cm} (18)

We will consider the situation when \( \delta < \omega_0 \) or \( R < 2\sqrt{L/C} \). The following notation was made:

\[ \omega_0^2 - \delta^2 = \omega'^2 \] \hspace{1cm} (19)

Consequently, the solving of the differential equation gives the following solutions:

\[ u(t) = E \left[ 1 - \frac{\omega_0}{\omega'} \cdot e^{\delta t} \cdot \sin(\omega' t + \beta') \right] \] \hspace{1cm} (20)

\[ i(t) = \frac{E}{\omega'L} e^{\delta t} \sin \omega' t \] \hspace{1cm} (21)
where

\[ \beta' = \arccos \frac{\delta}{\omega_0} \]  

(22)

4.2 SIMULINK model of the circuit

The SIMULINK model of the circuit after closing was done on grounds of the second order differential equation (12), which is put in the form (23) where the higher order derivative is separated:

\[ \frac{d^2 u}{dt^2} = \frac{1}{LC} \left[ E - RC \frac{du}{dt} \cdot u \right] \]  

(23)

The SIMULINK model from Fig. 22 generates the voltage capacitor and the current through the circuit during the transitory regime. To this purpose, two values of the R resistor are considered, which correspond to two important regimes:

- Aperiodic regime;
- Oscillatory regime.

For each regime the capacitor voltage and the circuit current variation diagrams are plotted.

Fig. 22. SIMULINK model of the circuit

If we consider:

- The value of the D.C. voltage: \( E = 24 \) [V],
- The circuit resistance: \( R = 1.5 \) [kΩ],
- The coil inductance: \( L = 10^{-4} \) [H],
- The capacitor capacity: \( C = 10 \) [nF],
we get the following MATLAB diagrams (Fig.23 and Fig.24) according to the equations (16) and (17) for a-periodic mode, and according to the equations (20) and (21) for oscillating mode (Fig.25 and Fig.26).

Fig. 23. The voltage capacitor variation in the a-periodic mode

Fig. 24. The current variation in the a-periodic mode

Fig. 25. The capacitor voltage variation in the oscillating regime
The k switcher is realized by using a voltage step signal which is applied at the terminal of the circuit, to simulate the closing of the k switcher. Changing the value of resistance, automatically leads to the updating of the voltage and current charts.

4.3. Analysis of inductive-capacitive circuits using the SimPowerSystems software package

The SimPowerSystems software package allows the study of the circuit behavior, when it is connected to a voltage step or a unitary impulse, using Continuous powergui block. The current variation in the circuit, the voltage variation of the capacitive element, or the frequency behavior of the circuit for different values of circuit elements can be studied.

4.3.1 SimPowerSystems model of the circuit

The MATLAB software package contains SimPowerSystems of Simulink, which can simulate the electrical circuits and analyze different operating regimes. Because the signal step response or pulse generates a transitory regime, the circuit behavior in these conditions can be studied. The simulation is shown in Fig.27:

The advantage of this simulation method is that the circuit can be tested at both signals, unit step and unitary impulse, and leads to several types of typical circuit diagrams. Two sets of values were chosen in this case for circuit parameters, which correspond to two special regimes.

In the simulation model from Fig.27 we used a Controlled Voltage Source controlled by a step voltage signal. In this way, the switcher closing can be simulated.

The SimPowerSystems software package allows the obtaining of a lot of diagrams for a circuit. So, we will study the capacitor voltage diagrams and current diagrams for the two occurring regimes:

- A-periodic regime:
- Oscillating regime.
Fig. 27. SimPowerSystems model of the circuit

1. **Voltage capacitor**

The a-periodic loading of the capacitor leads to the obtaining of the capacitor voltage variation diagrams when the unit step and the unitary impulse are applied to the input. These charts are presented below.

The following values of electrical parameters were considered:

- For a-periodic regime:
  - $E = 24 \, [V]$, the step voltage applied;
  - $R = 1.5 \, [k\Omega]$, the electrical circuit resistance;
  - $L = 10^{-4} \, [H]$, the circuit inductance;
  - $C = 10 \, [nF]$, the circuit capacity.
- For oscillating regime:
  - $E = 24 \, [V]$, the step voltage applied;
  - $R = 10 \, [\Omega]$, the electrical circuit resistance;
  - $L = 10^{-4} \, [H]$, the circuit inductance;
  - $C = 10 \, [nF]$, the circuit capacity.

To study the capacitor voltage variation, we follow the steps:

- Select by double clicking the *Continuous powergui* block from the simulation model;
- Select *Use LTI View* will appear in the new window;
- We will select U Voltage *Measurement*.

The POWERGUY analysis block, with the above values, allows the obtaining of the following Matlab diagrams (Fig.28 and Fig.29):
The POWERGUY block permits the analysis and plotting of the frequency characteristics of the capacitor voltage, where the frequency in logarithmic coordinates was considered on the horizontal axis (BODE diagram).

For this, we follow these steps:

- Select by double clicking *Continuous powerguy* block;
- Use LTI Viewer will appear in the window and we will select it;
- In the occurring window we will select U voltage Measurement;
- Now, we will select Edit / Plot Configuration / Bode

![LTI Viewer](image)

Fig. 30. Frequency diagrams of the voltage swing on the capacitive element

2. Study of circuit current

To study the circuit current variation the same procedure is used, mentioning that instead of selecting the U voltage Measurement, I Current Measurement must be selected.

A-periodic loading of the capacitor leads to obtaining the current variation diagrams when the unit step and the unitary impulse are applied to the input.

For this, we will follow these steps:
- Select Continuous powergui block diagram simulation by double click;
- Select Use LTI Viewer in the appeared window;
- Select I Current Measurement in the window appeared;
- Select Edit / Plot Configuration / Bode

The SimPowerSystems software package allows the obtaining of two diagrams for a circuit. So, we will study the current variation diagrams for the two occurring regimes:
- A-periodical regime:
- Oscillating regime.

The a-periodical and oscillating regime according to these values are obtained in the following charts (Fig.31 and Fig.32):
The POWERGUY block allows the analysis and plotting of frequency current characteristics, where the frequency in logarithmic coordinates was considered on the horizontal axis (BODE diagram). For this, we will follow these steps:

Fig. 31. The current variation for signal step and impulse in a-periodic mode

Fig. 32. The current variation for signal step and impulse in the oscillating regime
- Select Continuous powergui by double clicking the block diagram simulation;
- Select Use LTI Viewer in the appeared window;
- Select I Current Measurement in the window appeared;
- Select Edit / Plot Configuration / Bode

Fig. 33. Frequency diagrams of the current swing through the circuit

In the diagram from Fig.33 we can observe a maximum of current which correspond to a resonance regime. We can determine the resonance frequencies for an inductive-capacitive circuit using the simulation circuit model and the SimPowerSystems software package. With POWERGUY block, other electrical circuit parameters can be determined.

5. Conclusions

An important conclusion of this chapter is that the electrical circuits, regardless of configuration, can be studied with this modern method which involves the use of virtual medium.

Three elementary circuits are presented here, but the study modality can be extended to other circuits configuration as well.

If we consider an electric circuit, a first problem is that of writing the characteristic integral differential equations, which can be integrated with a simulation model and which uses the Simulink software package. These models can be conceived so that the electric circuit parameters must be electric input values in the simulation model. Thus, the obtained diagrams are automatically updated to any changes of circuit electrical parameters, and the influence of each parameter variation in the final diagrams can be studied.
In order to realize the simulating model which integrates the differential equations, the basic idea is to separate in the left member of equation, the higher order derivative. The analytical expressions obtained in the right member of the equation underlie the achievement of the simulating model by means of specific blocks of the virtual medium. In the case of complex circuits, the simulating model is conceived on a differential equations system.

Another given facility of this package programs, is the possibility to study the circuit behavior upon applying the standard signals (step unit, unitary impulse). The specific analysis block Continuous Powerguy allows the circuit response to these signals. It also allows the study of circuit frequency characteristics and the other specific diagrams: the coil hysteresis diagrams, line parameters, FTF analysis, Bode Diagrams, Nyquist diagrams, Nichols diagrams and others.

This study intends to be a starting point in the approach of more complex circuits.

6. References

Niculescu, T.; Pascaulescu, D.; Pana, L. (2010). *Study of the operating states of intrinsic safety Barriers of the electric equipment intended for use in atmospheres with explosion hazard*, Main Page of the Journal WSEAS TRANSACTIONS on CIRCUITS and SYSTEMS, ISSN: 1109-2734,


Building on MATLAB (the language of technical computing), Simulink provides a platform for engineers to plan, model, design, simulate, test and implement complex electromechanical, dynamic control, signal processing and communication systems. Simulink-Matlab combination is very useful for developing algorithms, GUI assisted creation of block diagrams and realisation of interactive simulation based designs. The eleven chapters of the book demonstrate the power and capabilities of Simulink to solve engineering problems with varied degree of complexity in the virtual environment.